

Soluții

$$1) a) \sqrt{11-6\sqrt{2}} = \sqrt{11-\sqrt{72}} = \sqrt{\frac{11+\sqrt{121-72}}{2}} - \sqrt{\frac{11-\sqrt{121-72}}{2}} = 3-\sqrt{2}$$

$$\sqrt{5-2\sqrt{6}} = \sqrt{5-\sqrt{24}} = \sqrt{\frac{5+\sqrt{25-24}}{2}} - \sqrt{\frac{5-\sqrt{25-24}}{2}} = \sqrt{3}-2$$

$$\Rightarrow \sqrt{3} + 3 - \sqrt{2} - \sqrt{3} + 2 = 5 - \sqrt{2}$$

$$b) a = (2^{\frac{2}{3}} \cdot 3^{\frac{1}{4}} \cdot 3^{\frac{1}{6}} \cdot 2^{-\frac{1}{3}}) \cdot (2^{\frac{1}{3}} \cdot 3^{-1} \cdot 3^{\frac{1}{2}}) = 3^{-1} \cdot 2^1 = \frac{2}{3} \in \mathbb{Q}$$

$$c) \sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}; \sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}; \sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$$

$$\Rightarrow \sqrt{2} < \sqrt[4]{5} < \sqrt[3]{4}$$

$$2) a) \log_2(25-7) - \log_2 9 = \log_2 \frac{18}{9} = \log_2 2 = 1$$

$$b) \log_{12} 18 = \frac{\log_3 18}{\log_3 12} = \frac{\log_3 9 + \log_3 2}{\log_3 2^2 + \log_3 3} = \frac{2+a}{2a+1}$$

$$c) \log(a+2b)^2 - \log 2^4 = \log(a \cdot b) \Leftrightarrow \log \frac{a^2+4ab+4b^2}{16} = \log ab$$

$$\Leftrightarrow a^2+4ab+4b^2 = 16ab \Leftrightarrow a+4b = 12ab$$

$$3) a) f(x) = y \text{ are sol} \Leftrightarrow \frac{2x}{x^2+1} = y \text{ are sol} \Leftrightarrow$$

$$\Leftrightarrow x^2 y - 2x + y = 0 \text{ are sol} \Leftrightarrow \Delta = 4 - 4y^2 \geq 0$$

$$\Leftrightarrow y \in [-1, 1] \Rightarrow \text{Im } f = [-1, 1]$$

$$b) \text{Cum } f(0) = f(2) = 1 \Rightarrow f \text{ nu e injectivă}$$

$$\text{Im}(f) = \text{Im}(f_1) \cup \text{Im}(f_2) = (-\infty, 3) \cup [0, \infty) = \mathbb{R}$$

$$\Rightarrow f \text{ surjectivă } (f_1(x) = 2x+1, f_2(x) = x-1)$$

$$c) \text{Fie } y \in (3, \infty). \text{ Căutăm } x \in (0, \infty) \text{ a.p. } f(x) = y$$

$$\Rightarrow 4^x + 2^x + 1 - y = 0 \Rightarrow x = \log_2 \frac{-1 + \sqrt{4y-3}}{2} \text{ sol unică}$$

$$\Rightarrow f \text{ bijectivă și } f^{-1}(y) = \log_2 \frac{-1 + \sqrt{4y-3}}{2} \in (0, \infty)$$

$$4) a) \frac{\sqrt{x+1}}{\sqrt{x-1}} = y \Rightarrow y + \frac{1}{y} = \frac{5}{2} \Rightarrow 2y^2 - 5y + 2 = 0$$

$$\Rightarrow y_1 = 2, y_2 = \frac{1}{2}. \text{ Din } \frac{\sqrt{x+1}}{\sqrt{x-1}} = 2 \Rightarrow x = \frac{5}{3}.$$

$$\text{Din } \frac{\sqrt{x+1}}{\sqrt{x-1}} = \frac{1}{2} \Rightarrow x = -\frac{5}{3} \text{ nu verifică cond. } x+1 > 0, x-1 > 0.$$

$$b) \log_2^2 x + 2 + \log_2 x = 4. \text{ Not. } \log_2 x = y, x > 0.$$

$$y^2 + y - 2 = 0 \Rightarrow y_1 = 1, y_2 = -2 \Rightarrow 1) \log_2 x = 1 \Rightarrow x = 2$$

$$2) \log_2 x = -2 \Rightarrow x = 2^{-2} = \frac{1}{4}$$

$$c) \text{Not. } 3^x = y, y > 0$$

$$\Rightarrow y^2 - y + m - 3 = 0, \text{ ec. ce trebuie să aibă } y_1, y_2 > 0.$$

$$\Leftrightarrow \Delta > 0, y_1 + y_2 > 0, y_1 \cdot y_2 > 0 \Leftrightarrow \begin{cases} 1 - 4m + 12 > 0 \\ 1 > 0; m - 3 > 0 \end{cases}$$

$$\Rightarrow m \in (3; \frac{13}{4}).$$