Soluții

(1) CE: $2x>0 \Rightarrow x>0$

$(\log_2 2+\log_2 x)^2+3(\log_2 4+\log_2 x)=13 \Rightarrow$ (notăm $\log_2 x=y$)

$(y+1)^2+3(2+y)=13 \Rightarrow y^2+5y-6=0 \Rightarrow y_1=1,\ y_2=-6$

$\Rightarrow \log_2 x=1 \iff x=2$ sau $\log_2 x=-6 \iff x=2^{-6}=\frac{1}{64}$.

(2) ε soluție a ec. $x^2+x+1=0 \Rightarrow \varepsilon^2+\varepsilon+1=0$ și $\varepsilon^3=1$.

$\Rightarrow (a+b\varepsilon+c\varepsilon^2)(a+b\varepsilon^2+c\varepsilon)=a^2+ab\varepsilon^2+ac\varepsilon+$
$+ab\varepsilon+b^2\varepsilon^3+bc\varepsilon^2+ac\varepsilon^2+bc\varepsilon^4+c^2\varepsilon^3=a^2+b^2+c^2+$
$+ab(\varepsilon^2+\varepsilon)+ac(\varepsilon^2+\varepsilon)+bc(\varepsilon^2+\varepsilon)=a^2+b^2+c^2-ab-ac-bc$
$=\frac{1}{2}\left[(a-b)^2+(a-c)^2+(b-c)^2\right]\geq 0$.

(3) $z=\dfrac{2\sin^2\frac{a}{2}-2i\sin\frac{a}{2}\cos\frac{a}{2}}{2\cos^2\frac{a}{2}+2i\sin\frac{a}{2}\cos\frac{a}{2}}=\dfrac{-2i\sin\frac{a}{2}\left(\cos\frac{a}{2}+i\sin\frac{a}{2}\right)}{2\cos\frac{a}{2}\left(\cos\frac{a}{2}+i\sin\frac{a}{2}\right)}=$

$=-i\,tg\frac{a}{2} \Rightarrow Re(z)=0$.

(4) Fie $MA\perp AB \Rightarrow m_{MA}\cdot m_{AB}=-1$

dar $m_{AB}=\dfrac{y_A-y_B}{x_A-x_B}=\dfrac{1}{3}$ $\Big| \Rightarrow m_{MA}=-3$

$\Rightarrow MA: y-y_A=m_{MA}(x-x_A) \Rightarrow y-2=-3(x-1) \iff$
$\iff 3x+y-5=0$

(5) $f(1)+f(2)=5$, $f(1),f(2)\in\{1,2,3,4,5,6,7,8\}$ $\Big| \Rightarrow (f(1),f(2))\in\{(1,4),(4,1),(2,3),(3,2)\}$

Mulțimea ordonată $(f(3),f(4),f(5))$ ~~este~~ submulțime în $\{1,2,3,4,5,6,7,8\}\setminus\{f(1),f(2)\} \Rightarrow$ poate fi aleasă în $A_6^3=120$ moduri

Cum $(f(1),f(2))$ poate fi aleasă în 4 moduri
$\Rightarrow 120\cdot 4=480$ funcții.

(6) $T_{k+1}=C_9^k\cdot(a^2)^{9-k}\cdot\left(\frac{1}{\sqrt[3]{a}}\right)^k=C_9^k\cdot a^{18-2k}\cdot a^{-\frac{k}{3}}=$

$=C_9^k\cdot a^{18-2k-\frac{k}{3}}=C_9^k\cdot a^{18-\frac{7k}{3}}$ conține pe $a^4 \iff 18-\frac{7k}{3}=4 \Rightarrow 14=\frac{7k}{3} \iff k=6$

$\Rightarrow T_{6+1}=T_7=C_9^6\cdot a^4$.